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On a class of second-order nonlinear difference equation

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Abstract

In this paper, we consider the rule of trajectory structure for a kind of second-order rational difference equation. With the change of the initial values, we find the successive lengths of positive and negative semicycles for oscillatory solutions of this equation, and the positive equilibrium point 1 of this equation is proved to be globally asymptotically stable.

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1 Introduction and preliminaries

Motivated by those work [1-17], especially [10], we consider in this paper the following second-order rational difference equation

$$x_{n+1} = \frac{1 + x_n^k x_{n-1}^l + a}{x_n^k + x_{n-1}^l + a}, n = -1, 0, 1, \ldots, \tag{1.1}$$

the initial values $x_{-1}, x_0 \in (0, +\infty)$, $a \in (0, +\infty)$ and $k, l \in (-\infty, +\infty)$.

Mainly, by analyzing the rule for the length of semicycle to occur successively, we describe clearly out the rule for the trajectory structure of its solutions and further derive the global asymptotic stability of positive equilibrium of Equation (1.1).

It is easy to see that the positive equilibrium $\bar{x}$ of Equation (1.1) satisfies

$$\bar{x} = \frac{1 + \bar{x}^{k+l} + a}{\bar{x}^k + \bar{x}^l + a}.$$

From this, we see that Equation (1.1) possesses a positive equilibrium $\bar{x} = 1$. In this paper, our work is only limited to positive equilibrium $\bar{x} = 1$.

Here, for readers' convenience, we give some corresponding definitions.

Definition 1.1. *A positive semicycle of a solution* $\{x_n\}_{n=-1}^{\infty}$ *of Equation (1.1) consists of a string of terms* $\{x_r, x_{r+1}, \ldots, x_m\}$*, all greater than or equal to the equilibrium* $\bar{x}$*, with* $r \geq -1$ *and* $m \leq \infty$ *such that*

$$\text{either } r = -1 \text{ or } r > -1 \text{ and } x_{r-1} < \bar{x}$$

and

$$\text{either } m = \infty \text{ or } m < \infty \text{ and } x_{m+1} < \bar{x}.$$

A negative semicycle of a solution $\{x_n\}_{n=-1}^{\infty}$ *of Equation (1.1) consists of a string of terms* $\{x_r, x_{r+1}, ..., x_m\}$, *all less than the equilibrium* $\bar{x}$, *with* $r \geq -1$ *and* $m \leq \infty$ *such that*

$$\text{either } r = -1 \text{ or } r > -1 \text{ and } x_{r-1} \geq \bar{x}$$

and

$$\text{either } m = \infty \text{ or } m < \infty \text{ and } x_{m+1} \geq \bar{x}.$$

The length of a semicycle is the number of the total terms contained in it.

Definition 1.2. *A solution* $\{x_n\}_{n=-1}^{\infty}$ *of Equation (1.1) is said to be eventually positive if* x_n *is eventually greater than* $\bar{x} = 1$. *A solution* $\{x_n\}_{n=-1}^{\infty}$ *of Equation (1.1) is said to be eventually negative if* x_n *is eventually smaller than* $\bar{x} = 1$.

Definition 1.3. *We can divide the solutions of Equation (1.1) into two kinds of types: trivial ones and nontrivial ones. A solution* $\{x_n\}_{n=-1}^{\infty}$ *of Equation (1.1) is said to be eventually trivial if* x_n *is eventually equal to* $\bar{x} = 1$; *otherwise, the solution is said to be nontrivial.*

If the solution is a nontrivial solution, then we can further divide the solution into two cases: non-oscillatory solution and oscillatory solution. A nontrivial solution $\{x_n\}_{n=-1}^{\infty}$ *of Equation (1.1) is regarded as non-oscillatory solution if* x_n *is eventually positive or negative; otherwise, the nontrivial solution is oscillatory.*

For the other concepts in this paper, see Refs.[1,2].

2 Trajectory structure rule

The solutions of Equation (1.1) include trivial ones, non-oscillatory ones and oscillatory ones, and their trajectory structure rule of the solutions is as follows.

2.1 Nontrivial solution

Theorem 2.1. *A positive solution* $\{x_n\}_{n=-1}^{\infty}$ *of Equation (1.1) is eventually trivial if and only if*

$$(x_{-1} - 1)(x_0 - 1) = 0. \tag{2.1}$$

Proof. Sufficiency. Assume that Equation (2.1) holds. Then according to Equation (1.1), we know that the following conclusions are true:

(i) If $x_{-1} = 1$, then $x_n = 1$ for $n \geq 1$.

(ii) If $x_0 = 1$, then $x_n = 1$ for $n \geq 1$.

Necessity. Conversely, assume that

$$(x_{-1} - 1)(x_0 - 1) \neq 0. \tag{2.2}$$

Then, we can show $x_n \neq 1$ for any $n \geq 1$. For the sake of contradiction, assume that for some $N \geq 1$,

$$x_N = 1 \text{ and that } x_n \neq 1 \text{ for any } -1 \leq n \leq N-1. \tag{2.3}$$

Clearly,

$$1 = x_N = \frac{1 + x_{N-1}^k x_{N-2}^l + a}{x_{N-1}^k + x_{N-2}^l + a}.$$

From this, we can know that

$$0 = x_N - 1 = \frac{(x_{N-1}^k - 1)(x_{N-2}^l - 1)}{x_{N-1}^k + x_{N-2}^l + a},$$

which implies $x_{N-1} = 1$, or $x_{N-2} = 1$. This contradicts with Equation (2.3).

Remark 2.2. *Theorem 2.1 actually demonstrates that a positive solution* $\{x_n\}_{n=-1}^{\infty}$*of Equation (1.1) is eventually nontrivial if* $(x_{-1} - 1)(x_0 - 1) \neq 0$*. So, if a solution is a nontrivial one, then* $x_n \neq 1$ *for any* $n \geq -1$.

2.2 Non-oscillatory solution

Lemma 2.3. *Let* $\{x_n\}_{n=-1}^{\infty}$*be a positive solution of Equation (1.1) which is not eventually equal to* 1*, then the following conclusion is true:*

(A) If $kl < 0$*, then* $(x_{n+1} - 1)(x_n - 1)(x_{n-1} - 1) < 0$*, for* $n \geq 0$;
(B) If $kl > 0$*, then* $(x_{n+1} - 1)(x_n - 1)(x_{n-1} - 1) > 0$*, for* $n \geq 0$;

Proof. First, we consider (A). According to Equation (1.1), we have that

$$x_{n+1} - 1 = \frac{(x_n^k - 1)(x_{n-1}^l - 1)}{x_n^k + x_{n-1}^l + a}, n = 0, 1, \ldots.$$

Considering $kl < 0$,

$$(x_{n+1} - 1)(x_n - 1)(x_{n-1} - 1) < 0.$$

Noting that $kl < 0$, that is $k \in (-\infty, 0)$ and $l \in (0, +\infty)$, or $k \in (0, +\infty\ -\infty, 0)$, and $l \in (-\infty, 0)$, one has $(x_n^k - 1)(x_n - 1) > 0$, $(x_{n-1}^l - 1)(x_{n-l} - 1) < 0$, or $(x_{n-1}^l - 1)(x_{n-l} - 1) > 0$, $(x_{n-1}^l - 1)(x_{n-l} - 1) > 0$. From those, one can get the result easily.

The proof of (B) is similar to (A).

Theorem 2.4. *Let* $kl < 0$*, there exist non-oscillatory solutions of Equation (1.1) with* $x_{-1}, x_0 \in (0, 1)$*, which must be eventually negative. There do not exist eventually positive non-oscillatory solutions of Equation (1.1).*

Proof. Consider a solution of Equation (1.1) with

$$x_{-1}, x_0 \in (0, 1).$$

We then know from Lemma 2.3 (A) that $0 < x_n < 1$ for $n \in N$, where $N \in 1, 2, 3, \ldots$. So, this solution is just a non-oscillatory solution and furthermore eventually negative.

Suppose that there exists eventually positive non-oscillatory of Equation (1.1). Then, there exists a positive integer N such that $x_n > 1$ for $n \geq N$. Thereout, for $n \geq N + 1$,

$$(x_{n+1} - 1)(x_n - 1)(x_{n-1} - 1) \geq 0.$$

This contradicts Lemma 2.3. So, there do not exist eventually positive non-oscillatory of Equation (1.1), as desired.

From Lemma 2.3 (B), we can get the result as follows, also.

Theorem 2.5. *Let* $kl > 0$*, there exist non-oscillatory solutions of Equation (1.1) with* $x_{-1}, x_0 \in (1, +\infty)$*, which must be eventually positive. There do not exist eventually negative non-oscillatory solutions of Equation (1.1).*

2.3 Oscillatory solution

Theorem 2.6. *Let* $kl < 0$, *and* $\{x_n\}_{-1}^{\infty}$*be a strictly oscillatory of Equation (1.1), then the rule for the lengths of positive and negative semicycles of this solution to occur successively is* ..., 2^+, 1^-, 2^+, 1^-, ...

Proof. By Lemma 2.3, one can see that the length of a negative semicycle is at most 3, and a positive semicycle is at most 2. On the basis of the strictly oscillatory character of the solution, we see that, for some integer $p \geq 0$, one of the following 32 cases must occur:

case 1: $x_p < 1$, $x_{p+1} < 1$;
case 2: $x_p > 1$, $x_{p+1} < 1$;
case 3: $x_p < 1$, $x_{p+1} > 1$;
case 4: $x_p > 1$, $x_{p+1} > 1$.
case 1 cannot occur. Otherwise, the solution is a non-oscillatory solution of Equation (1.1).

If Case 2 occurs, it follows from Lemma 2.3 that $x_{p+2} > 1$, $x_{p+3} > 1$, $x_{p+4} < 1$, $x_{p+5} > 1$, $x_{p+6} > 1$, $x_{p+7} < 1$, $x_{p+8} > 1$, $x_{p+9} > 1$, $x_{p+10} < 1$, ...

This means that rule for the lengths of positive and negative semicycles of the solution of Equation (1.1) to occur successively is ..., 2^+, 1^-, 2^+, 1^-, ... The proof for other cases, except Case 1, is completely similar to that of Case 2. So, the proof for this theorem is complete.

Theorem 2.7. *Let* $kl > 0$, *and* $\{x_n\}_{-1}^{\infty}$*be a strictly oscillatory of Equation (1.1), then the rule for the lengths of positive and negative semicycles of this solution to occur successively is* ..., 1^+, 2^-, 1^+, 2^-, ...

The proof of theorem (2.7) is similar to that of theorem (2.6).

3 Local asymptotic stability and global asymptotic stability

Before stating the oscillation and non-oscillation of solutions, we need the following key lemmas. For any integer a, denote $N_a = \{a, a + 1, ...,\}$.

3.1 Four Lemmas

Lemma 3.1. *Let* $k \in (0, 1]$, *and* $\{x_n\}_{n=-1}^{\infty}$*be a positive solution of Equation (1.1) which is not eventually equal to* 1, *then the following conclusions are valid:*

(a) $(x_{n+1} - x_n)(x_n - 1) < 0$, *for* $n \geq 0$;
(b) $(x_{n+1} - x_{n-1})(x_{n-1} - 1) < 0$, *for* $n \geq 0$.

Proof. First, we consider (a). From Equation (1.1), we obtain

$$x_{n+1} - x_n = \frac{1 - x_n^{k+1} + x_{n-1}^l x_n (x_n^{k-1} - 1) + a(1 - x_n)}{x_n^k + x_{n-1}^l + a},$$

From $k \in (0, 1]$ and $\{x_n\}_{n=-1}^{\infty}$ not eventually equal to 1, one can see that

$$(1 - x_n^{k+1})(1 - x_n) > 0, (1 - x_n^{1-k})(1 - x_n) \geq 0, x_n^k + x_{n-1}^l > 0.$$

This teaches us that $(x_{n+1} - x_n)(1 - x_n) > 0$, $n = 0, 1, ...$ That is to say, $(x_{n+1} - x_n)(x_n - 1) < 0$, $n = 0, 1, ...$ So, the proof of (a) is complete.

Second, one investigates (b). From Equation (1.1), one has

$$x_{n+1} - x_{n-1} = \frac{1 - x_n^k x_{n-1} + x_{n-1}^l (x_n^k - x_{n-1}) + a(1 - x_n)}{x_n^k + x_{n-1}^l + a}, \tag{3.1}$$

From Equation (1.1), one gets

$$1 - x_n x_{n-1}^{\frac{1}{k}} = \frac{x_{n-1}^k \left(1 - x_{n-1}^{\frac{1}{k^2}}\right)}{x_{n-1}^k + x_{n-2}^l + a}, \tag{3.2}$$

According to $k \in (0, 1]$ and $\{x_n\}_{n=-1}^{\infty}$ not eventually equal to 1, one arrives at

$$\left(1 - x_{n-1}^{\frac{1}{k^2}}\right)(1 - x_{n-1}) \geq 0. \tag{3.3}$$

From Equations (3.2) and (3.3), we know $\left(1 - x_n x_{n-1}^{\frac{1}{k}}\right)(1 - x_{n-1}) > 0$. So, we can get immediately

$$\left(1 - x_n^k x_{n-1}\right)(1 - x_{n-1}) > 0. \tag{3.4}$$

From Equation (1.1), one can have

$$x_n - x_{n-1}^{\frac{1}{k}} = \frac{x_{n-1}^{k+l} \left(1 - x_{n-1}^{\frac{1}{k^2}}\right)}{x_{n-1}^k + x_{n-2}^l + a}, \tag{3.5}$$

According to $k \in (0, 1]$ and $\{x_n\}_{n=-1}^{\infty}$ not eventually equal to 1, one arrives at

$$\left(1 - x_{n-1}^{\frac{1}{k^2}}\right)(1 - x_{n-1}) \geq 0. \tag{3.6}$$

From Equations (3.5), (3.6), we can obtain that $\left(x_n - x_{n-1}^{\frac{1}{k}}\right)(1 - x_{n-1}) > 0$, i.e.,

$$\left(x_n^k - x_{n-1}\right)(1 - x_{n-1}) > 0. \tag{3.7}$$

By virtue of Equations (3.1), (3.4), (3.7), we see that (b) is true.

The proof for Lemma (3.1) is complete.

Lemma 3.2. *Let* $\{x_n\}_{n=-1}^{\infty}$ *be a positive solution of Equation (1) which is not eventually equal to* 1, *then* $(x_{n+1} - x_{n-2})(x_{n-2} - 1) < 0$, *for* $n \geq 1$.

Proof. By virtue of Equation (1.1), one gets

$$x_{n+1} - x_{n-2} = \frac{(1 - x_n^k x_{n-2}) + (x_n^k - x_{n-2}) x_{n-1}^l + a(1 - x_{n-2})}{x_n^k + x_{n-1}^l + a}, n = 0, 1, \ldots. \tag{3.8}$$

By virtue of Equation (1.1), one obtains that

$$x_{n-1} - x_{n-2}^{\frac{1}{k^2}} = \frac{\left(1 - x_{n-2}^{\frac{k^3+1}{k^2}}\right) + a\left(1 - x_{n-2}^{\frac{1}{k^2}}\right) + x_{n-3}^l x_{n-2}^k \left(1 - x_{n-2}^{\frac{1}{k^3}}\right)}{x_{n-2}^k + x_{n-3}^l + a}. \tag{3.9}$$

According to $k \in (0, 1]$ and $\{x_n\}_{n=-1}^{\infty}$ not eventually equal to 1, we get

$$\left(1 - x_{n-2}^{\frac{k^3+1}{k^2}}\right)(1 - x_{n-2}) > 0, \left(1 - x_{n-2}^{\frac{1}{k^2}}\right)(1 - x_{n-2}) > 0, \left(1 - x_{n-2}^{\frac{1}{k^3}}\right)(1 - x_{n-2}) > 0.$$

So,

$$\left(x_{n-1} - x_{n-2}^{\frac{1}{k^2}}\right)(1 - x_{n-2}) > 0. \tag{3.10}$$

That is

$$\left(x_{n-1}^{k} - x_{n-2}^{\frac{1}{k}}\right)(1 - x_{n-2}) > 0. \tag{3.11}$$

By virtue of Equation (1.1), we can know

$$1 - x_n x_{n-2}^{\frac{1}{k}} = \frac{\left(x_{n-1}^{k} - x_{n-2}^{\frac{1}{k}}\right) + x_{n-2}^{l}\left(1 - x_{n-1}^{k+\frac{1}{k}}\right) + a\left(1 - x_{n-2}^{\frac{1}{k}}\right)}{x_{n-1}^{k} + x_{n-2}^{l} + a}. \tag{3.12}$$

Utilizing (3.11),(3.12), adding $\left(1 - x_{n-1}^{k+\frac{1}{k}}\right)(1 - x_{n-2}) > 0, \left(1 - x_{n-2}^{\frac{1}{k}}\right)(1 - x_{n-2}) > 0$ when $k \in (0, 1]$, we know the following is true

$$\left(1 - x_n x_{n-2}^{\frac{1}{k}}\right)(1 - x_{n-2}) > 0.$$

So,

$$\left(1 - x_n^{k} x_{n-2}\right)(1 - x_{n-2}) > 0. \tag{3.13}$$

Similar to (3.13), we know this is true

$$\left(x_n - x_{n-2}^{\frac{1}{k}}\right)(1 - x_{n-2}) > 0.$$

So,

$$\left(x_n^{k} - x_{n-2}\right)(1 - x_{n-2}) > 0. \tag{3.14}$$

From (3.8),(3.13)and (3.14), one obtains that the following is true

$$(x_n - x_{n-2})(1 - x_{n-2}) > 0.$$

This shows Lemma (3.2) is true.

Lemma 3.3. *Let* $x_{-1}, x_0 \in (0, 1)$, *then the following conclusions are true:*

(a) If $l > 0$ *and* $-1 < k < 0$ *or* $l < 0$ *and* $0 < k < 1$, *then* $(x_{n+1} - x_n) < 0$, *for* $n \geq 0$;

(b) If $k > 0$ *and* $-1 < l < 0$ *or* $k < 0$ *and* $0 < l < 1$, *then* $(x_{n+1} - x_{n-1}) < 0$, *for* $n \geq 0$.

The proof of lemma (3.3) can be completed by Equation (1.1), theorem 2.4 and properties of power function easily.

Lemma 3.4. *Let* $x_{-1}, x_0 \in (1, \infty)$, *then the following conclusions are true:*

(a) If $l > 0$ and $0 < k < 1$ or $l < 0$ and $-1 < k < 0$, then $(x_{n+1} - x_n) < 0$, for $n \geq 0$;

(b) If $k > 0$ and $0 < l < 1$ or $k < 0$ and $-1 < l < 0$, then $(x_{n+1} - x_{n-1}) < 0$, for $n \geq 0$.

The proof of lemma (3.4) can be completed by Equation (1.1), theorem 2.5 and properties of power function easily.

First, we consider the local asymptotic stability for unique positive equilibrium point $\bar{x}$ of Equation (1.1). We have the following results.

3.2 Local asymptotic stability

Theorem 3.5. *The positive equilibrium point of Equation (1.1) is locally asymptotically stable.*

Proof. The linearized equation of Equation (1.1) about the positive equilibrium point $\bar{x}$ is

$$y_{n+1} = 0 \cdot y_n + 0 \cdot y_{n-1}, n = 0, 1, \ldots,$$

and so it is clear from the paper [[2], Remark 1.3.7] that the positive equilibrium point $\bar{x}$ of Equation (1.1) is locally asymptotically stable. The proof is complete.

We are now in a position to study the global asymptotically stability of positive equilibrium point $\bar{x}$.

3.3 Global asymptotic stability of oscillatory solution

Theorem 3.6. *The positive equilibrium point of Equation (1.1) is globally asymptotically stable when $k \in (0, 1]$ and $l \in (0, +\infty)$.*

Proof We must prove that the positive equilibrium point $\bar{x}$ of Equation (1.1) is both locally asymptotically stable and globally attractive. Theorem 3.5 has shown the local asymptotic stability of $\bar{x}$. Hence, it remains to verify that every positive solution $\{x_n\}_{n=-1}^{\infty}$ of Equation (1.1) converges to $\bar{x}$ as $n \to \infty$. Namely, we want to prove

$$\lim_{n\to\infty} x_n = \bar{x} = 1. \tag{3.15}$$

Consider now $\{x_n\}$ to be non-oscillatory about the positive equilibrium point $\bar{x}$ of Equation (1.1). By virtue of Lemma 3.1(a), it follows that the solution is monotonic and bounded. So, $\lim_{n\to\infty} x_n$ exists and is finite. Taking limits on both sides of Equation (1.1), one can easily see that (3.15) holds.

Now let $\{x_n\}$ be strictly oscillatory about the positive equilibrium point of Equation (1.1). By virtue of Theorem 2.6, one understands that the rule for the lengths of positive and negative semicycles occurring successively is ..., 2^+, 1^-, 2^+, 1^-, 2^+, 1^-, ... For simplicity, for some nonnegative integer p, we denote by $\{x_p, x_{p+1}\}^+$ the terms of a positive semicycle of length two, followed by $\{x_{p+2}\}^-$, a negative semicycle with semicycle length one, then a positive semicycle of length two and a negative semicycle of length one, and so on. Namely, the rule for the lengths of positive and negative semicycles to occur successively can be periodically expressed as follows:

$$\{x_{p+3n}, x_{p+3n+1}\}^+, \{x_{p+3n+2}\}^-, \{x_{p+3n+3}, x_{p+3n+4}\}^+, \{x_{p+3n+5}\}^-, n = 0, 1, 2, \ldots.$$

Lemma (3.1) (a), (b) and Lemma (3.2) teaches us that the following results are true:

(A) $x_{p+3n} > x_{p+3n+1} > x_{p+3n+3} > x_{p+3n+4}$, $n = 0, 1, 2, \ldots$

(B) $x_{p+3n+2} < x_{p+3n+5} < x_{p+3n+8}$, $n = 0, 1, 2, \ldots$

So, from (A) one can see that $\{x_{p+3n}\}_{n=0}^{\infty}$ is decreasing with lower bound 1. So, the limit $S = \lim_{n\to\infty} x_{p+3n}$ exists and is finite.

Furthermore, From (A) one can further obtain

$$S = \lim_{n\to\infty} x_{p+3n+1}$$

Similarly, by (B) one can see that $\{x_{p+3n+2}\}_{n=0}^{\infty}$ is increasing with upper bound 1. So, the limit $T = \lim_{n\to\infty} x_{p+3n+2}$ exists and is finite.

Now, it suffices to prove $S = T = 1$.

Noting that

$$x_{p+3n+2} = \frac{1 + x_{p+3n+1}^k x_{p+3n}^l + a}{x_{p+3n+1}^k + x_{p+3n}^l + a}, \tag{3.16}$$

$$x_{p+3n+3} = \frac{1 + x_{p+3n+2}^k x_{p+3n+1}^l + a}{x_{p+3n+2}^k + x_{p+3n+1}^l + a}, \tag{3.17}$$

Taking limits on both sides of the Equations (3.16) and (3.17), respectively, we get

$$T = \frac{s^{k+l} + 1 + a}{s^k + s^l + a}, \tag{3.18}$$

$$S = \frac{s^k + T^l + 1 + a}{s^k T^l + a}, \tag{3.19}$$

From this one can see $S = 1$. Again, by Equation (3.18), we have $T = 1$, too. These show that (3.15) is true. The proof for Theorem 3.6 is complete.

Theorem 3.7. *The positive equilibrium point of Equation (1.1) is globally asymptotically stable when* $k \in (0, 1]$ *and* $l \in (-\infty, 0)$.

The proof of theorem 3.7 is similar to that of theorem 3.6 by virtue of theorem 3.5, theorem 2.7, Lemma (3.1), Lemma (3.2) and Equation (1.1).

3.4 Global asymptotic stability of non-oscillatory solution

Theorem 3.8. *The positive equilibrium point of Equation (1.1) is globally asymptotically stable when* $x_{-1}, x_0 \in (0, 1)$ *and one of the following conditions is satisfied:*

(a) $-1 < k < 0$ *and* $l > 0$;
(b) $0 < k < 1$ *and* $l < 0$;
(c) $k > 0$ *and* $-1 < l < 0$;
(d) $k < 0$ *and* $0 < l < 1$.

The proof of theorem 3.8 is similar to that of theorem 3.6 by virtue of theorem 2.4, theorem 3.5, Lemma (3.3) and Equation (1.1).

Theorem 3.9. *The positive equilibrium point of Equation (1.1) is globally asymptotically stable when* $x_{-1}, x_0 \in (1, +\infty)$ *and one of the following conditions is satisfied:*

(a) $-1 < k < 0$ *and* $l < 0$;
(b) $0 < k < 1$ *and* $l > 0$;
(c) $k < 0$ *and* $-1 < l < 0$;

(d) $k > 0$ *and* $0 < l < 1$.

The proof of theorem 3.9 is similar to that of theorem 3.6 by virtue of theorem 2.5, theorem 3.5, Lemma (3.4) and Equation (1.1).

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Authors' contributions

All authors carried out the proof. All authors conceived of the study and participated in its design and coordination. All authors read and approved the final manuscript.

Competing interests

The authors declare that they have no competing interests.

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